To conclude this year’s iteration of Biology 182, Dr. H and I will undertake a discussion of climate change. Here follow some notes that will help you get up to speed.

**Modeling the Earth’s Temperature.**

The simplest climate models (e.g., Covey, 1989) identify temperatures, \( T_a, T_b, \ldots \), at which incoming (solar radiation) and outgoing (infrared) fluxes balance. These *energy balance models* (EBMs) assume spatial homogeneity – think of the earth as stationary and surrounded by tanning lamps – and further, that radiative fluxes are uniquely specified by temperature. The latter assumption ignores the fact that the response of real planets to changes in flux entails *time lags* – for example, the time required for glaciers to melt. As a result, these models are said to be *quasistatic*.

Energy gain, \( G(T) \), and loss, \( L(T) \), functions are shown in Figure 1. Gain results from the absorption of short-wave radiation – principally light – from the sun; loss, from the re-radiation of solar inputs as heat. Of course, not all the incoming energy is absorbed. Some is reflected back out into space before it can warm the earth. The fraction so reflected is the planetary *albedo*, \( \alpha(T) \), hence the relation,

\[
G(T) = k(1 - \alpha(T)),
\]

where \( k \) is the solar “constant.”\(^1\) Note that the \( \alpha(T) \) is assumed to be in equilibrium with planetary temperature, \( T \).\(^2\) The albedo of ice being something in excess of 0.9, and that of water and soil on the order of 0.5, \( G(T) \) increases sharply at \( T = 0^\circ \) C.

Energy loss – in the form of long-wave radiation (heat) – is also presumed to depend on temperature. One approach is to assume black-body radiation,

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1. As discussed later, solar flux varies on a range of time scales, the most familiar of which is the roughly ten-year sunspot cycle.

2. This is the quasistatic assumption: one imagines that \( T \) changes slowly so that processes such as the formation and melting of glaciers can track it.
\[ L(T) = \sigma T^4, \quad (2a) \]
where \( \sigma \) is the Stefan-Boltzmann constant. Alternatively, one can assume a linear relation, e.g.,
\[ L(T) = 204 + 2.17T \quad (2b) \]
(Budyko’s approximation). Equation (2b) is empirical – it gives reasonable results for for \( 0 \leq T \leq 30^\circ \) C. It’s advantage over Equation (2a) – which follows from physical law – is that it implicitly incorporates the effect of greenhouse warming by water vapor.

**Optional.** Recalling that heat capacity, \( C \), converts flux into temperature, we arrive at
\[ C \frac{dT}{dt} = \frac{k}{4} \left[ 1 - \alpha(T) \right] - L(T), \quad (3) \]
where \( k/4 \) is solar flux averaged over the earth’s surface.

As shown in Figure 1, \( G(T) \) and \( L(T) \) can intersect up to three times. The corresponding temperatures, \( T_a, T_b \) and \( T_c \), are equilibria. Of these, \( T_a \) and \( T_c \) are stable, while \( T_b \), is unstable. Note that \( T_a \) corresponds to a frozen earth – all water in the form of ice; \( T_c \), to an ice-free planet.

**Latitudinal and Seasonal Variation.** Equations (1)-(3) specify what is sometimes referred to as a “zero-dimensional” model because there is no explicit spatial variation. The next step up in model complexity is a one-dimensional EBM, wherein the earth is divided into latitudinal bands (Figure 2). As in the zero-dimensional case, energy from the sun enters as short-wave radiation and is re-radiated as infrared. But now, there is the added possibility of exchange between adjacent bands as shown in Figure 2. Additionally, averaged values of solar input and albedo are replaced by latitudinal functions.

**Optional.** For each latitudinal band, we replace Equation (3) with
\[ C \frac{dT(\theta)}{dt} = A(\theta)[k(\theta)(1 - \alpha(T(\theta))) - L(T(\theta))] - \nabla Q \quad (4) \]
where \( \nabla Q \) is the heat flux between bands. Transport between bands proceeds by advection, mixing, and differences in atmospheric pressure. Often, these processes are modeled by Fickian diffusion, \( Q = const(\partial T / \partial x) \), where \( x \) is the spatial component – in this case, latitude.
Equations such as (4) are said to be one-dimensional because there is a single, spatial dimension. Obviously, they are more realistic than zero-dimensional models and, in fact, contain the minimum information required for explicitly modeling latitudinal effects. This having been said, it is still possible to use zero-dimensional models to obtain a qualitative understanding of what transpires along a latitudinal transect. To see how, we note that the flux of incident solar radiation declines with latitude. As a result, \( G(T) \) is greatest at the equator and least at the poles. If we imagine that Figure 1 depicts the situation for mid-latitudes, we can obtain the situation shown in Figure 3 – a single stable equilibrium temperature, \( T_a \) (frozen earth), at the poles and a single equilibrium, \( T_c \) (no ice), at the equator. In terms of Figure 1, \( T_a \) and \( T_b \) approach each other and eventually collide as one moves toward the equator. Likewise, as one moves poleward, \( T_b \) and \( T_c \) collide. The points of collision are called bifurcations, and the colliding equilibria are annihilated.

In like fashion, one can model seasonal variations in climate at mid-latitudes. Now the upper gain curve in Figure 3 corresponds to summer, and the lower curve, to winter. As shown in Figure 4, this sets up what is called a hysteresis loop. Here, we plot the three equilibria as functions of varying insolation. As the days lengthen, insolation increases, and temperature tracks \( T_a \), which slowly increases. At the same time, \( T_b \) declines. Eventually, \( T_a \) and \( T_b \) collide, at which point, the temperature increases abruptly to \( T_c \). After the summer solstice, insolation declines. Now the system tracks \( T_c \) until the latter collides with \( T_b \), at which point the temperature drops back to \( T_a \). The significance of hysteresis is that there is a delay in the system’s response to changing insolation. As such, it explains why the first day of spring is generally cooler than the first day of fall. Note that this time lag has nothing to do with the sorts of factors discussed on the evening weather report – movement of air masses, changes in circulation, etc., factors for which there is no provision zero-dimensional EBM. What the lag does reflect is the existence of multiple equilibria that pop into existence and / or are destroyed at different temperatures.

**Greenhouse Gases.** Greenhouse gases warm the earth by reflecting a portion of re-radiated infrared back to the planet’s surface. The principal greenhouse gas is water vapor – which is why nighttime temperatures stay high during the Tucson summer once the monsoon gets going, and
humidity rises. Other, less important, greenhouse gases are CH4 and CO2. It is worth emphasizing that the greenhouse effect is what makes earth habitable. Absent greenhouse gases, “snowball earth” [REF] would be a frigid, and contemporary, reality. Of course, as everyone knows – but see below – CO2 concentrations have been steadily increasing as the result of human activity – hence “anthropogenic warming” and what some imagine to be the crisis that looms. As in the case of latitudinal variation, accurate modeling of increasing [CO2] concentrations requires something more complicated than a zero-dimensional EBM. But again we can obtain qualitative understanding of the phenomenon by playing with the curves in Figure 1. Specifically, we observe that reducing the loss curve, $L(T)$, increases $T_a$ and reduces $T_b$. If the effect is sufficiently severe, the equilibria annihilate each other leaving only the no-ice equilibrium as shown in Figure 5. As in the case of the seasonal cycle, varying the concentration [GHG] of greenhouse gases can set up a hysteresis loop. Replace “Insolation” with “[GHG]” in Figure 4, and you get the picture. To use a phrase that has become popular with climate warming enthusiasts, there is “tipping point.” Pump enough CO2 into the atmosphere and you everywhere abolish $T_a$. To recover the frozen wastes would then require reductions in [CO2] beyond the point at which the climate shifted. This is the so-called “consensus view,” the evidence for which, we describe below.